

# 2023 VCE Mathematical Methods 2 external assessment report

## General comments

This is the first year of the new study design and most students were able to respond effectively to the questions involving the introduced concepts, such as Newton's method in Questions 3f. and 3g. and the point of inflection in Question 3d.

As the examination papers were scanned, students needed to use a HB or darker pencil or a dark blue or black pen.

Some students had difficulty understanding some of the concepts, such as the difference between average value of a function and average rate of change (Questions 2b. and 2c.). This year two questions involved strictly increasing and strictly decreasing functions. In Question 3e. many students used round brackets instead of square brackets. In Question 5b. either bracket type was appropriate as the maximal domain was not required. Some students did not attempt Question 3a. They appeared to be confused by the limit notation  $\lim_{x \rightarrow \infty} g(x)$ .

Students need to ensure that they show adequate working where a question is worth more than one mark, as communication is important in mathematics. A number of questions could be answered using trial and error in this year's paper, especially in relation to probability, Questions 4f. and 4g. Drawing a diagram can often be helpful to show the output for some of the trials. Students are allowed to use their technology to find the area between two curves using the bounded area function, but they must show some relevant working if the question is worth more than one mark. Often, questions require that a definite integral is written down, such as in Question 1cii. In Question 4d. students were expected to identify and write down the  $n$  and  $p$  values for the binomial distribution, not just the answer.

There were a number of transcription errors, and incorrect use of brackets and vinculums, in Questions 1b., 1ci., 1d. and 3ci. Students needed to take more care when reading the output from their technology. For

example, some students wrote  $-\frac{\sqrt{7}+1}{3}$  instead of  $\frac{-\sqrt{7}+1}{3}$  in Question 1b. Others wrote  $x = \pm \frac{\sqrt{5}-1}{2}$

instead of  $x = \frac{\pm\sqrt{5}-1}{2}$  in Question 1ci.

Students need to make sure they give their answers to the required accuracy. In most of Question 1, Questions 3b. and 3h., and Question 4i., exact values were required. In all questions where a numerical answer is required, an exact value must be given unless otherwise specified. A number of students gave approximate answers as their final response to these questions. In Question 3f. some students gave their answers correct to two decimal places when the response required three decimal places.

There were a number of rounding errors, especially in Questions 1ciii., Questions 3d. and 3f., Questions 4a. and 4h., and Question 5ci. Students must make sure they have their technology set to the correct float or take more care when reading and transcribing the output.

Students need to improve their communication with 'show that' and transformation questions. There was only one 'show that' question this year, Question 2a. Sufficient working out, presented as a set of logical steps with a conclusion, needed to be shown. In Question 5a., the transformation question, the correct wording needed to be used, such as 'reflect in the  $y$ -axis'.

In general, students appeared to have made good use of their technology, for example in finding the equations of tangent lines, finding bounded areas and using graph sliders to get approximate answers to complicated questions. Some students, however, need more practice at interpreting the output from their technology, especially when the technology uses numerical methods to find solutions. In Question 3cii., the tangent line passes through the origin, but some students gave  $y = 4.255x + 8.14E - 10$  as their final answer, not appearing to recognise that  $8.14E - 10$  should be zero.

Most students made a good attempt at the probability questions. Some students, however, still misinterpreted the wording. Errors occurred in Questions 4a., b., c. and d.

Students are reminded to read questions carefully before responding and then to reread questions after they have answered them to ensure that they have given the required response. Question 1a. required the answers to be in coordinate form. Question 3ci. required an equation. In Question 3cii. many students only found the value of  $a$  and did not continue to find the equation of the tangent.

## Specific information

### Section A – Multiple-choice questions

The table indicates the percentage of students who chose each option. Grey shading indicates the correct response. The statistics in this report may be subject to rounding, resulting in a total of more or less than 100 per cent.

Question	Correct answer	% A	% B	% C	% D	% E	% N/A	Comments
1	E	2	16	2	3	77	0	
2	A	53	29	3	6	8	0	
3	E	22	12	11	7	47	0	The domain of $p$ is $[-2,3)$ and the domain of $q$ is $(-1,5]$ . The domain of the sum function $p+q$ is the intersection of the two domains. $[-2,3) \cap (-1,5]$ $=(-1,3)$
4	B	15	55	10	14	4	1	
5	D	9	16	8	62	4	0	
6	D	3	4	41	49	2	0	$\int_3^{10} f(x)dx = C$ and $\int_7^{10} f(x)dx = D$ $\int_3^{10} f(x)dx = \int_3^7 f(x)dx + \int_7^{10} f(x)dx$ $C = \int_3^7 f(x)dx + D$

Question	Correct answer	% A	% B	% C	% D	% E	% N/A	Comments
								$C - D = \int_3^7 f(x) dx$ $\int_7^3 f(x) dx = D - C$
7	C	3	12	60	7	17	0	
8	C	12	14	49	16	9	1	<p>Let <math>G</math> represent a green ball being selected.</p> $\Pr(G \geq 1)$ $= 1 - \Pr(G = 0)$ $= 1 - \left(\frac{m}{n+m}\right)^8$
9	C	7	9	42	31	10	0	$f(x) = \begin{cases} \tan\left(\frac{x}{2}\right) & 4 \leq x < 2\pi \\ \sin(ax) & 2\pi \leq x \leq 8 \end{cases}$ <p>For <math>f</math> to be continuous at <math>x = 2\pi</math>,</p> $\tan\left(\frac{x}{2}\right) = \sin(ax)$ <p>For <math>f</math> to be smooth at <math>x = 2\pi</math>,</p> $\frac{d}{dx} \tan\left(\frac{x}{2}\right) = \frac{d}{dx} \sin(ax)$ <p>So, solving <math>\tan\left(\frac{x}{2}\right) = \sin(ax)</math> and</p> $\frac{d}{dx} \tan\left(\frac{x}{2}\right) = \frac{d}{dx} \sin(ax) \text{ for } a,$ $a = -\frac{1}{2}$
10	B	8	60	14	13	5	1	
11	E	6	13	8	51	22	1	$\frac{d}{dx}[f(x)g(x)]$ $= f(x)g'(x) + g(x)f'(x)$ <p>at <math>x = -2</math></p> $= f(-2)g'(-2) + g(-2)f'(-2)$ $= (-7 \times 2) + (3 \times 8)$ $= 10$
12	E	11	14	26	18	29	1	<p>From observation, <math>k \geq 0</math> and the maximum will occur when <math>k = 0</math>, <math>E(X) = 2</math>.</p> $E(X) = -k^2 + k - 2k^2 - 8k + 2$ $= -3k^2 - 7k + 2$

Question	Correct answer	% A	% B	% C	% D	% E	% N/A	Comments								
								When $k = 0$ , $E(X) = 2$ .								
13	C	16	22	52	7	3	1									
14	D	6	33	22	29	10	1	$y = x(3x - 1)(x + 3)(x + 1)$ The positive $x$ -intercept is $\frac{1}{3}$ . Find the tangent line at $x = a$ . $y_T = (12a^3 + 33a^2 + 10a - 3)x - a^2(9a^2 + 22a + 5)$ Solve $y_T\left(\frac{1}{3}\right) = 0$ for $a$ . $a = \frac{-\sqrt{7} - 4}{3}$ , $a = \frac{\sqrt{7} - 4}{3}$ or $a = \frac{1}{3}$ Hence there are three solutions. Alternatively, a graphical approach could be taken.								
15	A	65	6	17	4	6	0									
16	B	14	69	8	5	4	0									
17	B	6	28	26	26	13	1	The base of the cylinder has a circumference of $2\pi r$ units. $y = 2\pi r$ , $r = \frac{y}{2\pi}$ $h = x - 4r$ , $h = x - \frac{2y}{\pi}$ The formula for volume of the cylinder is $V = \pi r^2 h$ $V = \pi \left(\frac{y}{2\pi}\right)^2 \left(x - \frac{2y}{\pi}\right) = \frac{\pi xy^2 - 2y^3}{4\pi^2}$								
18	E	12	9	22	26	29	1	$f: [-a\pi, a\pi] \rightarrow \mathbb{R}, f(x) = \sin(ax)$ <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th></th> <th>Number of local minima</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>1</td> </tr> <tr> <td>2</td> <td>4</td> </tr> <tr> <td>3</td> <td>9</td> </tr> </tbody> </table> The number of local minima is $a^2$ .		Number of local minima	1	1	2	4	3	9
	Number of local minima															
1	1															
2	4															
3	9															
19	D	27	11	13	32	16	1	$x^2 + (4k + 3)x + 4k^2 - \frac{9}{4} = 0$ $\Delta = (4k + 3)^2 - 4\left(4k^2 - \frac{9}{4}\right) > 0$ for two unique solutions.								

Question	Correct answer	% A	% B	% C	% D	% E	% N/A	Comments
								$k > -\frac{3}{4}$ One solution has to be positive and the other negative. Solve $x^2 + (4k + 3)x + 4k^2 - \frac{9}{4} = 0$ for $k$ , when $x = 0$ and $k > -\frac{3}{4}$ . $k = \frac{3}{4}$ $-\frac{3}{4} < k < \frac{3}{4}$ OR Use the quadratic formula and solve: $\frac{-b + \sqrt{b^2 - 4ac}}{2a} > 0$ and $\frac{-b - \sqrt{b^2 - 4ac}}{2a} < 0$ $\frac{-4k - 3 + \sqrt{(4k + 3)^2 - 4\left(4k^2 - \frac{9}{4}\right)}}{2} > 0$ So solve $\frac{-4k - 3 - \sqrt{(4k + 3)^2 - 4\left(4k^2 - \frac{9}{4}\right)}}{2} < 0$ for $k$ $-\frac{3}{4} < k < \frac{3}{4}$
20	A	30	19	26	14	9	1	$f(x) = \log_e\left(x + \frac{1}{\sqrt{2}}\right)$ , $g(x) = \sin(x)$ where $x \in (-\infty, 5)$ $(f \circ g)(x) = \log_e\left(\sin(x) + \frac{1}{\sqrt{2}}\right)$ $\sin(x) + \frac{1}{\sqrt{2}} > 0$ $\sin(x) = -\frac{1}{\sqrt{2}}$ , $x = \dots -\frac{\pi}{4}, \frac{5\pi}{4}$ as $x < 5$ $x \in \left(-\frac{\pi}{4} + 2\pi k, \frac{5\pi}{4} + 2\pi k\right)$ , $k \in \mathbb{Z}^- \cup \{0\}$ $(g \circ f)(x) = \sin\left(\log_e\left(x + \frac{1}{\sqrt{2}}\right)\right)$ $\log_e\left(x + \frac{1}{\sqrt{2}}\right) < 5$ , $x = e^5 - \frac{1}{\sqrt{2}}$ $x \in \left(-\frac{1}{\sqrt{2}}, e^5 - \frac{1}{\sqrt{2}}\right)$

Question	Correct answer	% A	% B	% C	% D	% E	% N/A	Comments
								<p>The largest interval of <math>x</math> values for which <math>(f \circ g)(x)</math> and <math>(g \circ f)(x)</math> both exist is</p> $\left(-\frac{\pi}{4} + 2\pi k, \frac{5\pi}{4} + 2\pi k\right) \cap \left(-\frac{1}{\sqrt{2}}, e^5 - \frac{1}{\sqrt{2}}\right),$ $k \in \mathbb{Z} \cup \{0\}$ $= \left(-\frac{1}{\sqrt{2}}, \frac{5\pi}{4}\right)$

## Section B

### Question 1a.

Marks	0	1	Average
%	9	91	0.9

$(-1, 0)$ ,  $(0, 0)$  and  $(2, 0)$

This question was answered well. Coordinates were required.

### Question 1b.

Marks	0	1	2	Average
%	6	25	68	1.6

Solve  $f'(x) = 0$

$$\left(\frac{-\sqrt{7}+1}{3}, \frac{2(7\sqrt{7}-10)}{27}\right) \text{ and } \left(\frac{\sqrt{7}+1}{3}, \frac{-2(7\sqrt{7}+10)}{27}\right)$$

Some students only gave the  $x$  values. Exact answers were required.

### Question 1ci.

Marks	0	1	Average
%	14	86	0.9

$$x = 2, x = \frac{-1 \pm \sqrt{5}}{2}$$

Some students incorrectly transcribed  $x = \frac{-1 \pm \sqrt{5}}{2}$  from their technology, giving the answer  $x = \pm \frac{\sqrt{5}-1}{2}$ .

Exact answers were required.

## Question 1cii.

Marks	0	1	2	Average
%	25	15	61	1.4

$$\int_{\frac{-\sqrt{5}-1}{2}}^{\frac{\sqrt{5}-1}{2}} (f(x) - g(x)) dx + \int_{\frac{\sqrt{5}-1}{2}}^2 (g(x) - f(x)) dx \text{ or } \int_{\frac{-(\sqrt{5}+1)}{2}}^2 |f(x) - g(x)| dx$$

Some students only gave one of the definite integrals.  $\int_{\frac{-\sqrt{5}-1}{2}}^2 (f(x) - g(x)) dx$  was a common incorrect answer.

There were a lot of sign errors, where students were subtracting the equations the wrong way around. Some unsuccessfully split the integrals into extra parts.

## Question 1ciii.

Marks	0	1	Average
%	42	58	0.6

5.95

Students who set up the definite integrals correctly in part cii. were generally successful with this question. 5.94 was a common incorrect answer.

## Question 1d.

Marks	0	1	2	3	4	Average
%	61	11	9	6	13	1.0

Method 1 (equating coefficients),  $(x - a)(x - b)^2 = f(x) + k$ ,  
 $x^3 - (a + 2b)x^2 + (2ab + b^2)x - ab^2 = x^3 - x^2 - 2x + k$ ,  $-(a + 2b) = -1$ ,  $2ab + b^2 = -2$ ,

$$a = \frac{-2\sqrt{7} + 1}{3}, b = \frac{\sqrt{7} + 1}{3}, \text{ or } a = \frac{2\sqrt{7} + 1}{3}, b = \frac{-\sqrt{7} + 1}{3}$$

Method 2 (using transformations), turning point is on the x-axis, solve  $(x - a)(x - b)^2 = f(x) + k$  for a when

$$k = \frac{2(7\sqrt{7} + 10)}{27}, b = \frac{\sqrt{7} + 1}{3} \text{ and when } k = \frac{-2(7\sqrt{7} - 10)}{27}, b = \frac{-\sqrt{7} + 1}{3}$$

$$a = \frac{-2\sqrt{7} + 1}{3}, b = \frac{\sqrt{7} + 1}{3}, \text{ or } a = \frac{2\sqrt{7} + 1}{3}, b = \frac{-\sqrt{7} + 1}{3}$$

This question was not done well. There were many different approaches taken. Those who used method 1 were generally successful. Those who used method 2 often had sign errors in their expressions for  $k$ . Exact answers were required. Some students only gave one set of values for  $a$  and  $b$ .

## Question 2a.

Marks	0	1	2	Average
%	16	18	66	1.5

$$\text{Period} = \frac{2\pi}{b} = 30, \quad b = \frac{\pi}{15}, \quad h(0) = 15, \quad -60 + c = 15, \quad c = 75$$

As this was a 'show that' question, appropriate working with logical sequencing needed to be shown. Many students were able to show that  $c = 75$ . Some students wrote  $\text{Period} = b$  instead of  $\text{Period} = \frac{2\pi}{b}$ .

## Question 2b.

Marks	0	1	2	Average
%	54	3	42	0.9

$$\frac{1}{7.5} \int_0^{7.5} h(t) dt = 36.80$$

Some students had the correct formula for average value of a function but used incorrect values.  $\frac{1}{60} \int_0^{60} h(t) dt$  was often seen. Other students found the average rate of change.

## Question 2c.

Marks	0	1	Average
%	45	55	0.5

$$\frac{h(7.5) - h(0)}{7.5} = 8$$

A common incorrect answer was  $-8$ .

## Question 2di.

Marks	0	1	Average
%	60	40	0.4

$$k = 135, \quad m = 2$$

Many students were able to find  $k$ , but not  $m$ .  $m = \frac{1}{2}$  was often seen.

### Question 2dii.

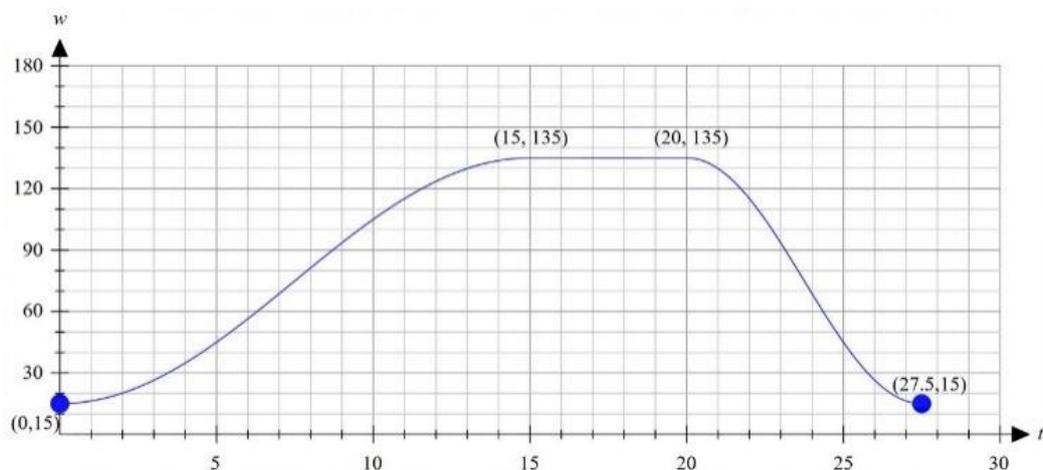
Marks	0	1	2	Average
%	76	12	12	0.4

$$w(20) = 135, \quad n = 5 + 30p, \quad p \in \mathbb{Z}$$

This question was not done well. Some students were able to set up a correct equation. A general solution was required. Some students wrote  $p \in \mathbb{R}$ .

### Question 2diii.

Marks	0	1	2	3	Average
%	39	25	12	24	1.2



Many students had the correct graph for  $0 \leq t \leq 15$ . The coordinates of the endpoints were missing on some graphs. Some students did not draw graphs with the correct curvature. Linear graphs were sometimes seen.

### Question 3a.

Marks	0	1	Average
%	25	75	0.7

5

This question was done well. Some students did not attempt the question and appear not to have recognised the notation  $\lim_{x \rightarrow \infty} g(x)$ . A common incorrect answer was 6.

### Question 3b.

Marks	0	1	Average
%	15	85	0.9

$$\log_e(2) \text{ or } \ln(2)$$

This question was done well. Some students did not include the base.

### Question 3ci.

Marks	0	1	Average
%	48	52	0.5

$$y = 2^x \log_2(2)x - (a \log_2(2) - 1) \times 2^x + 5 \text{ or } y = 2^a \log_e(2)x - a2^a \log_e(2) + 2^a + 5$$

An equation was required. There were many transcription errors such as  $y = 2^a \ln(2) - 2^a a \ln(2) + 2a - 5$  and  $y = 2^a \ln(2) - 2^a \ln(2) + 2^a + 5$ . Some students attempted to find the equation by hand, making algebraic errors.

### Question 3cii.

Marks	0	1	2	Average
%	52	34	15	0.6

$$0 = -a2^a \log_e(2) + 2^a + 5, a = 2.61784\dots, y = 4.255x$$

Some students did not substitute  $(0, 0)$  into the correct equation. Many misread the question and found the equation of the tangent line at  $x = 0$ , giving  $y = 0.693x + 6$  as the answer. Some substituted  $a = 0$  rather than  $x = 0$  into their equation.  $y = 4.255x + 8.14E - 10$  was often seen.

### Question 3d.

Marks	0	1	Average
%	42	58	0.6

$$(2.06, -0.07)$$

Many students gave the coordinates of the stationary points  $(0.49, 1.16)$  and  $(3.21, -1.05)$  rather than the coordinates of the point of inflection. There were some rounding errors.

### Question 3e.

Marks	0	1	Average
%	65	35	0.4

$$[0.49, 3.21]$$

Round brackets were often seen; these were incorrect as the largest interval of  $x$  values was required, which included the interval endpoints. In some cases, it was impossible to determine whether the student meant round or square brackets. Another incorrect response was  $(-\infty, 0.49] \cup [3.21, \infty)$ . These students have incorrectly interpreted the question requirements as asking for intervals where the function is strictly increasing.

### Question 3f.

Marks	0	1	2	Average
%	36	10	54	1.2

$$x_1 = -1.443, x_2 = -0.897, x_3 = -0.773$$

Many students were familiar with Newton's method. Answers were required to three decimal places. Some students only had one correct answer. Others had rounding errors.

### Question 3g.

Marks	0	1	Average
%	79	21	0.2

The solutions to  $\log_e(2) \times 2^x - 2x = 0$  will give the  $x$  values of the turning points of the graph. The tangents

to the graph will be horizontal lines and  $h'(x) = 0$ . Hence,  $x_{n+1} = x_n - \frac{h(x)}{h'(x)}$  will be undefined.

There were some good explanations. Some students only mentioned the two solutions.

### Question 3h.

Marks	0	1	2	Average
%	86	12	3	0.2

$$f(x) = 0 \text{ and } f'(x) = 0, n = e$$

This question was not done well. Many students indicated that  $f'(x) = 0$  but did not combine it with  $f(x) = 0$ . Some formulated the question correctly but did not provide an answer. Others found an approximate value for the answer such as  $n = 2.7$ . An exact answer was required.

### Question 4a.

Marks	0	1	Average
%	21	79	0.8

$$0.1587$$

This question was answered well. There were some rounding errors. 0.1586 was sometimes seen. Some students found  $\Pr(D < 6.8)$  rather than  $\Pr(D > 6.8)$ .

### Question 4b.

Marks	0	1	Average
%	41	59	0.6

$$6.83$$

A common error was that students solved  $\Pr(D > a) = 0.9$ , giving  $a = 6.57$  as the answer.

### Question 4c.

Marks	0	1	Average
%	23	77	0.8

0.9938

This question was answered well.  $\Pr(D \leq 6.94) \approx 0.9918$  was seen occasionally. Students need to be aware that with continuous probability  $\Pr(D < 6.95) = \Pr(D \leq 6.95)$ . Some students gave 0.0062 as the answer, which was the probability of the tennis ball being larger than 6.95 cm.

### Question 4d.

Marks	0	1	2	Average
%	28	18	54	1.3

$X \sim \text{Bi}(4, 0.99379\dots)$ ,  $\Pr(X \geq 3) = 0.9998$

Some students gave only the answer. Appropriate working must be shown for questions worth more than one mark. Students needed to give the  $n$  and  $p$  values. Some students solved  $\Pr(X = 3)$  or  $\Pr(X > 3)$ .

### Question 4e.

Marks	0	1	2	Average
%	31	16	53	1.2

$$\Pr(6.54 < D < 6.86 | D < 6.95) = \frac{\Pr(6.54 < D < 6.86)}{\Pr(D < 6.95)} = \frac{0.89040\dots}{0.99379\dots} = 0.8960$$

Most students realised this was a conditional probability question.  $\frac{0.89040\dots}{0.99977\dots} = 0.8906\dots$  was a common

incorrect answer.  $\frac{\Pr(6.54 < D < 6.86) \times \Pr(D < 6.95)}{\Pr(D < 6.95)} = 0.8904\dots$  was also a common incorrect approach.

### Question 4f.

Marks	0	1	2	Average
%	59	15	26	0.7

$$\frac{6.86 - 6.7}{\sigma} = 2.5758\dots \text{ or } \Pr(6.54 < D < 6.86) = 0.99 \text{ or } \Pr\left(\frac{6.54 - 6.7}{\sigma} < Z < \frac{6.86 - 6.7}{\sigma}\right) = 0.99, \sigma = 0.06$$

The maximum value of the standard deviation was not asked for in the question. Hence  $0 < \sigma \leq 0.06$  and  $\sigma = 0.00, 0.01, 0.02, 0.03, 0.04, 0.05$  or  $0.06$  were accepted.

$\Pr(D < 6.86) = 0.99$ ,  $\frac{6.86 - 6.7}{\sigma} = 2.3263\dots$  was a common incorrect approach. Trial and error could be used but students must make sure they show some appropriate working. Drawing a diagram and showing the probabilities was acceptable.

## Question 4g.

Marks	0	1	2	Average
%	66	15	19	0.5

$$\hat{p} = \frac{0.7382 + 0.9493}{2} = 0.84375, z = \sqrt{\frac{0.84375 \times (1 - 0.84375)}{32}} = 0.10555, z = 1.6444,$$

$$\Pr(-1.6444... < Z < 1.6444...) = 0.8999..., 90\%$$

Many students calculated  $\hat{p}$  incorrectly, and  $\hat{p} = 0.8904$  was often seen. Some had the correct  $z$  value but then gave the answer as 95%.

## Question 4h.

Marks	0	1	Average
%	43	57	0.6

$$0.1345$$

This question was answered well. There were some rounding errors.

## Question 4i.

Marks	0	1	Average
%	45	55	0.6

$$3(\pi^2 + 4)$$

This question was answered reasonably well. An exact answer was required.

## Question 4j.

Marks	0	1	2	Average
%	89	5	6	0.2

Method 1 (transform the mean), to maintain area of 1,  $a = \frac{1}{b}$  or  $b = \frac{E(W)}{E(V)}$ ,  $a = \frac{3}{2}$ ,  $b = \frac{2}{3}$

Method 2 (simultaneous equations),  $\int_{30b}^{(3\pi^2+30)b} g(w)dw = 1$  and  $\int_{30b}^{(3\pi^2+30)b} w \times g(w)dw = 2\pi^2 + 8$ ,  $a = \frac{3}{2}$ ,  $b = \frac{2}{3}$

This question was not done well. Some students did not attempt the question. Others were able to recognise that  $a = \frac{1}{b}$  but were unable to find their values. A common incorrect answer was  $a = \frac{2}{3}$  and  $b = 1$ . Many of those who attempted the second method did not multiply the terminals by  $b$ .

## Question 5a.

Marks	0	1	2	Average
%	18	48	35	1.2

Reflect in the  $y$ -axis and then translate 2 units to the right; or translate 2 units left and then reflect in the  $y$ -axis; or translate 2 units to the right only, as  $f$  is an even function.

This question was done reasonably well. The order of the transformations needed to be correct, as well as the wording. A common incorrect answer was 'reflect in the  $y$ -axis and then translate 2 units to the left'.

## Question 5b.

Marks	0	1	2	Average
%	40	21	39	1.0

A domain of  $[1, \infty)$ , with a range of  $[2, \infty)$ ; or a domain of  $(1, \infty)$ , with a range of  $(2, \infty)$ ; or, if  $g_1$  was defined with a domain that is a subset of  $[2, \infty)$ , then the domain and range of the student's  $g_1^{-1}$  was accepted because the question did not ask for the maximal domains.

Often the domain and range were reversed. Students gave the domain and range of  $g_1$ , not  $g_1^{-1}$ .

Many students appeared to find the notation confusing.

## Question 5ci.

Marks	0	1	Average
%	45	55	0.6

$P(1.27, 1.27)$ ,  $Q(4.09, 4.09)$

This question was answered well. Some students did not realise that  $P$  and  $Q$  were points on the line  $y=x$  and had different values for  $x$  and  $y$ . Others substituted their  $x$  values back into the equation to find  $y$  and made rounding errors.  $Q(4.09, 4.10)$  was sometimes seen.

## Question 5cii.

Marks	0	1	2	Average
%	65	6	29	0.6

$$2 \int_{1.27\dots}^{4.09\dots} (x - g(x)) dx = 5.56$$

Some students set up the correct definite integral but did not provide an answer. Others did not multiply the

definite integral by 2, giving 2.78 as the answer. Some used  $\int_{1.27\dots}^{4.09\dots} (g_1^{-1} - g(x)) dx$ , which is incorrect. Other

students had the correct answer but set up the definite integral incorrectly.

### Question 5d.

Marks	0	1	Average
%	87	13	0.1

$$n = -1$$

This question was not done well. A common incorrect answer was  $n = 1$ .

### Question 5e.

Marks	0	1	Average
%	96	4	0.0

$$1.27$$

This question was not done well. Many students did not attempt the question.

### Question 5f.

Marks	0	1	2	Average
%	86	2	12	0.3

$$A = \int_{1.45091\dots}^{8.78157\dots} h_1^{-1}(x) - h(x) dx = 43.91$$

There were some good attempts at this question. Those students who set up the correct definite integral generally gave the correct answer. Some students had incorrect terminals of integration. A common incorrect terminal was  $x = 2.468$ , which is the  $x$  value of the point of intersection of the graphs of  $h$  and  $y = x$ , not  $h_1^{-1}$  and  $h$ .